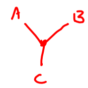
Monday, March 05, 2012
8:28 AM

Mr. Feynman and Mr. Lagrangian

Recall the ABC's:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi_A\partial^\mu\phi_A - \frac{1}{2}\left(\frac{m_A c}{\hbar}\right)^2\phi_A^2 + \frac{1}{2}\partial_\mu\phi_B\partial^\mu\phi_B - \frac{1}{2}\left(\frac{m_B c}{\hbar}\right)^2\phi_B^2 + \frac{1}{2}\partial_\mu\phi_C\partial^\mu\phi_C - \frac{1}{2}\left(\frac{m_C c}{\hbar}\right)^2\phi_C^2 - g\,\phi_A\phi_B\phi_C$$

In evaluating diagrams built from — A, — B, — C w/  we used: $-ig$ vertex factor

$\frac{i}{q_j^2 - m_j^2c^2}$ virtual particle propagators

There is actually a systematic way to extract the Feynman rules for a given Lagrangian, but to derive it requires QFT.
We will simply state the results.

		ABC example
Vertex factors:	Write $i\mathcal{L}_{int}$ in momentum space ($i\hbar\partial_\mu \to p_\mu$)	$-ig\,\phi_A\phi_B\phi_C$
	Erase the field variables	$-ig$
Propagators:	Write the relevant free-particle e.o.m. in momentum space	$\partial_\mu\partial^\mu\phi + \left(\frac{mc}{\hbar}\right)^2\phi = 0 \Rightarrow [p^2 - (mc)^2]\phi = 0$
	Multiply the inverse of the term in brackets $\times i$	$\frac{i}{p^2 - m^2c^2}$

Who's house?! Dirac's house!

Consider the QED Lagrangian:

$$\mathcal{L} = i\hbar c\bar{\psi}\gamma^\mu\partial_\mu\psi - mc^2\bar{\psi}\psi - q\bar{\psi}\gamma^\mu\psi A_\mu - \frac{1}{16\pi}(\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu)$$

The fundamental vertex is 

or any other charged particle

and the vertex factor we get from $i\mathcal{L}_{int} = -iq\bar{\psi}\gamma^\mu\psi A_\mu$ removing $\bar{\psi}, \psi, \sqrt{\frac{\hbar c}{4\pi}}A_\mu$ giving: $-i\sqrt{\frac{4\pi}{\hbar c}}\,q\gamma^\mu \equiv ig_e\gamma^\mu$

For the propagators we can have both virtual photons and electrons/positrons.

Electrons/Positrons we use: $i\gamma^\mu\partial_\mu\psi - \frac{mc}{\hbar}\psi = 0 \Rightarrow [\gamma^\mu p_\mu - mc]\psi = 0 \Rightarrow \dfrac{i(\gamma^\mu p_\mu + mc)}{p^2 - m^2c^2}$

For photons: Start w/ the massive Proca equation: $\partial_\mu(\partial^\mu A^\nu - \partial^\nu A^\mu) + (\frac{mc}{\hbar})^2 A^\nu = 0$

$$\partial_\mu \to -\tfrac{i}{\hbar}p_\mu \qquad -\tfrac{i}{\hbar}p_\mu\left(-\tfrac{i}{\hbar}p^\mu A^\nu + \tfrac{i}{\hbar}p^\nu A^\mu\right) + (\tfrac{mc}{\hbar})^2 A^\nu = 0$$

We want this as one thing operating on A^ν so let's massage it a bit. First get rid of $\hbar$ and i:

$$-p_\mu p^\mu A^\nu + p_\mu p^\nu A^\mu + (mc)^2 A^\nu = 0$$

Multiply by $\eta_{\lambda\nu}$ on both sides.

$$\eta_{\lambda\nu}(-p^2 A^\nu + p_\mu p^\nu A^\mu) + (mc)^2\eta_{\lambda\nu}A^\nu = 0$$

$$\eta_{\lambda\nu}(-p^2 + (mc)^2)A^\nu + \underbrace{p_\mu p_\lambda A^\mu}_{\text{rename } \mu\to\nu \text{ since repeated}} = 0$$

$$[\eta_{\lambda\nu}(-p^2 + (mc)^2) + p_\nu p_\lambda]A^\nu = 0$$

Everywhere replace $\lambda \to \mu$

$$[\eta_{\mu\nu}(-p^2 + (mc)^2) + p_\mu p_\nu]A^\nu = 0$$

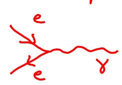
For $m \neq 0$ the propagator is: $i[\;]^{-1} = \underbrace{\dfrac{-i}{p^2 - (mc)^2}\left[\eta_{\mu\nu} - \dfrac{p_\mu p_\nu}{(mc)^2}\right]}_{\text{Useful for weak interactions later}}$

For $m = 0$ we also need transversality, i.e. $p_\nu A^\nu = 0$ leaving

$$[-\eta_{\mu\nu}p^2]A^\nu \Rightarrow i[\;]^{-1} = \frac{-i\eta_{\mu\nu}^{-1}}{p^2} = \underbrace{\frac{-i\eta_{\mu\nu}}{p^2}}_{\text{Photon propagator}}$$

Monday, March 24, 2014
1:14 PM

QED (for now just charged leptons e, μ, τ and photons γ)

Diagrams are built from: 

(or μ, τ versions)

In the ABC theory, all d.o.f. were scalars so: a) order was unimportant
b) we were guaranteed M would be a scalar

In QED e, μ, τ are spinors and γ is a vector so: a) order is important
b) we have to be careful to ensure M is a scalar

One new complication is that in most experiments we used unpolarized in-states and sum over all out states.
This means in calculating $|M|^2$ we must a) average over incoming spin states
b) sum over outgoing spin states

Useful expressions:	Electron	Positron	Photon
Wavefunction	$\psi(x) = a e^{-\frac{i}{\hbar} p\cdot x} u^{(s)}(p)$	$\psi(x) = a e^{\frac{i}{\hbar} p\cdot x} v^{(s)}(p)$	$A_\mu(x) = a e^{-\frac{i}{\hbar} p\cdot x} \epsilon_\mu^{(s)}$ $\quad s=1,2$
E.o.M. (mom. space)	$(\gamma^\mu p_\mu - mc)u = 0$ DE	$(\gamma^\mu p_\mu + mc)v = 0$ DE	$p^\mu \epsilon_\mu = 0$ LC, $\epsilon^0 = 0$ CG
Adjoint	$\bar{u} = u^\dagger \gamma^0$	$\bar{v} = v^\dagger \gamma^0$	$\epsilon^{\mu *}$
Adjoint E.o.M.	$\bar{u}(\gamma^\mu p_\mu - mc) = 0$	$\bar{v}(\gamma^\mu p_\mu + mc) = 0$	$p_\mu \epsilon^{\mu *} = 0$
Orthonormality	$\bar{u}^{(s)} u^{(s')} = 2mc\,\delta_{ss'}$	$\bar{v}^{(s)} v^{(s')} = -2mc\,\delta_{ss'}$	$\epsilon_\mu^{(s)} \epsilon^{\mu * (s')} = -\delta_{ss'}$
Completeness	$\sum_s u^{(s)} \bar{u}^{(s)} = \gamma^\mu p_\mu + mc$	$\sum_s v^{(s)} \bar{v}^{(s)} = \gamma^\mu p_\mu - mc$	$\sum_s \epsilon_i^{(s)} \epsilon_j^{(s)*} = \delta_{ij} - \hat{p}_i \hat{p}_j$

The Dirac spinors for $\vec{p} \neq 0$ in the new convention are:

$$u^{(1)} = \sqrt{\frac{E+mc^2}{c}} \begin{pmatrix} 1 \\ 0 \\ \frac{c p_z}{E+mc^2} \\ \frac{c(p_x + i p_y)}{E+mc^2} \end{pmatrix} \qquad u^{(2)} = \sqrt{\frac{E+mc^2}{c}} \begin{pmatrix} 0 \\ 1 \\ \frac{c(p_x - i p_y)}{E+mc^2} \\ \frac{-c p_z}{E+mc^2} \end{pmatrix}$$

$$v^{(1)} = \sqrt{\frac{E+mc^2}{c}} \begin{pmatrix} \frac{c(p_x - i p_y)}{E+mc^2} \\ \frac{-c p_z}{E+mc^2} \\ 0 \\ 1 \end{pmatrix} \qquad v^{(2)} = -\sqrt{\frac{E+mc^2}{c}} \begin{pmatrix} \frac{c p_z}{E+mc^2} \\ \frac{c(p_x + i p_y)}{E+mc^2} \\ 1 \\ 0 \end{pmatrix}$$

And for the spinor matrices:

$$\gamma^0 = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

w/ $\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$